STRUCTURAL MECHANICS LAB REPORT: PIN-JOINTED FRAME

Sara MOTWANI
1st year student MEng Engineering and Architectural Design
Material, Mechanics & Making Course – Structural Mechanics
Academic Year 2020 - 2021

ABSTRACT

The Pin-Jointed Frame experiment deals with a framework based on 30° and 60° angles. Through this lab it is aimed to evaluate the forces and strains in the members of a pin jointed frame. The strain in the members of the frame are subjected to a variety of loading conditions and these are related to the forces carried by the members. Furthermore, the actual values obtained from performing the experiment are compared with the theoretical values obtained via manual calculation using the method of joints. Based on the observations made through the course of this experiment the data is analysed and compared with the hypothesis made prior to starting the experiment. After observing the data obtained as a result of the experiment, the hypothesis is confirmed or rejected as well as key observations from the data are noted. The learnings from the entire experiment are assessed in conclusion.

Keywords: Pin-Jointed Frame; Strain; Loading Conditions; Method of Joints; Hypothesis.

1. INTRODUCTION

Trusses are stable frameworks used widely in the field of structural engineering for its stability and ability to withstand large loads without breaking down. A Pin-Jointed Frame is a truss framework wherein the members are inclined at 30° and 60° angles. In this experiment, each member of the framework has a sensor bonded to its surface that measures the strain in the members. These sensors are called strain gauges and work on the principle of experiencing a change in electrical resistance when they are stressed or compressed. The members are joined using the special joint pieces and nuts and bolts. This Pin-Jointed Framework is mounted to a pin joint on one end and a roller joint on the other end. Using the apparatus, forces of varied magnitudes are applied to two different points of the framework and results are recorded and observed from the Digital Strain display. These results are then analysed and compared to the hypothesis made prior to starting the experiment.

2. METHOD

The experiment is carried out in a laboratory using specially designed teaching equipment manufactured by TecQuipment. This equipment is precision engineered. An electronic load cell is used to apply loads onto the pin-jointed frame, while the Digital Force Display electronically measures and displays this force during the experiment. The strain is measured using the strain gauges. The Digital Strain display shows all member strains read in microstrain (με). This works under the assumption that the members operate in their elastic range.

2.1 Apparatus

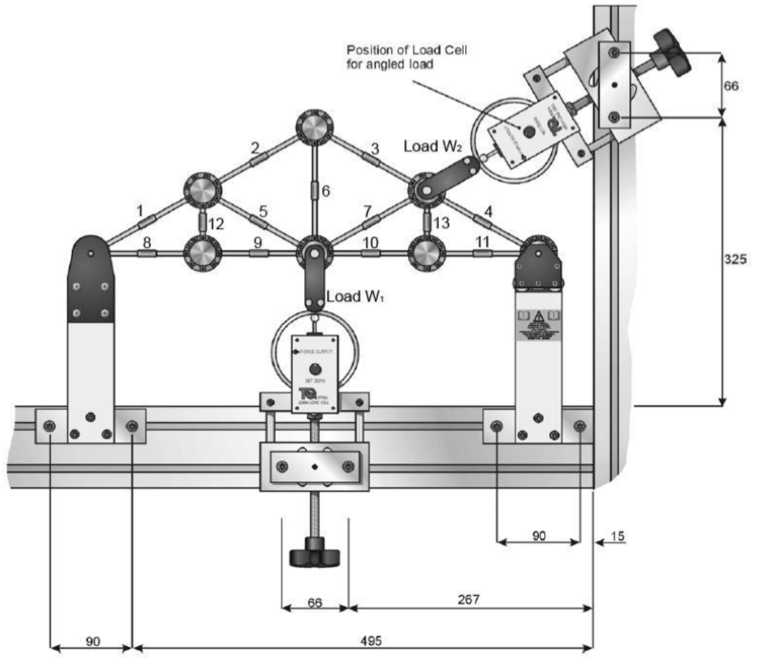


Figure 1. Apparatus of Pin-Jointed Truss used in the experiment

As shown above, the apparatus used is a Pin-Jointed Frame Truss from TecQuipment, the STR8 with members having length of 140 mm and diameter of cross section as 6 mm. Below the truss and to the top right are Load Cells that are used to apply forces onto the truss. The truss is supported by a pin support on the left and a roller support on the right. The structures seen bonded to the center of each member are the strain gauges. All these results are measured and displayed on electronic displays during the experiment.

2.2 Formulae Used

According to Hooke's Law, The Young's Modulus (E) is given by:

$$E = \sigma / \varepsilon \tag{1}$$

The Stress (σ) is defined by:

$$\sigma = F / A \tag{2}$$

Where:
E = Young's modulus (N/mm2)
σ = Stress in the member (N/mm2)
ε = Displayed strain

F = Force in member;
A = Cross-sectional area of member.

Combining and rearranging equations (1) and (2) the following relationship between Applied Force and the Strain can be obtained:

$$F = \varepsilon \,.\, E \,.\, A \tag{3}$$

3. HYPOTHESIS

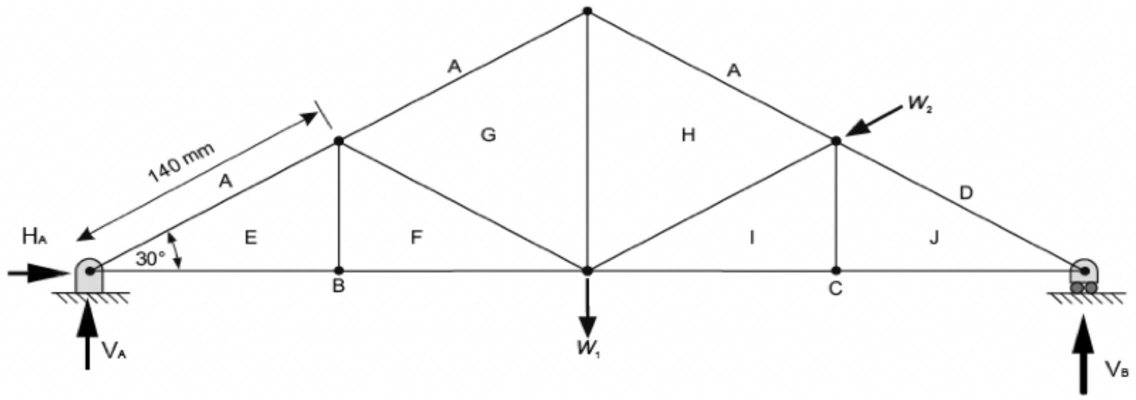


Figure 2. Idealised Roof Truss and Reactions, Showing Member Identifiers

In this experiment the aim is to find the weak points of the frame and so the strain of the various members of the frame is the principal behavior that is being tested when the member is subjected to a variety of loading conditions to relate this to the forces carried. This behavior is tested by subjecting the frame to loads of different magnitudes and to loads at different angles/points on the frame. It is to be noted however, that the members of the truss have the same material and area of cross section through the course of the experiment.

It can be predicted that the basic parameters that will govern the stress and strains in the truss are the magnitude as well as the direction of the applied force as is evident from the formula's for stress and strain that their values depend on the force. Apart from the force, the cross-sectional area of the member is also a parameter as the stress varies inversely with it and the type of material used (it's Youngs Modulus) is also a parameter as strain varies inversely with it. In addition to these parameters, it is also likely that the shape of the truss, which included the length of the members along with their orientation and quantity, also might be a factor to govern the stress and strain.
It can be hypothesised that in this experiment, as the applied load is increased, the strain in the members shall also increase and that too linearly. This hypothesis can be based off the fact that from the formulae from stress and strain it is deduced that the strain is directly proportional to the applied force.

The experimental results obtained from the experimental apparatus can be expected to be roughly the same as the values calculated theoretically. It is not possible for the values to be exactly the same due to the many assumptions made in calculation as well as the factors like changing temperatures and experimental errors.

The members that appear to be in tension are GH, BE, BF, CI and CJ. This can be assumed because, if these members were to be removed the structure would collapse outwards under a horizontal force indicating that they were in tension. Similarly, the members that appear to be in compression are AE, AG, AH and DJ. This can be assumed because, if these members were to be removed the structure would collapse inwards under a horizontal force. In addition to this, it also appears that members EF, FG, IH and IJ are not in tension or compression, i.e., they experience no strain. This is assumed because, if these members were to be removed there would be no collapsing of the structure. However, these are likely to change when the force is applied at an angle as it is evident that the point of application of force as well as its orientation changes. Hence applying the same principle as mentioned above, the members likely to be in tension are GH, CI and CJ, in compression are AE, AG, AH, DJ, CI, CJ and IH and the zero members are EF, FG and IJ.

4. CALCULATION OF FORCES

4.1 Experimental Force

The Experimental Force is calculated using the equation (3).
Diameter = 6 mm
Area = π (dia/2)^2

⇨ Area = 28.27 mm^2

E = 210000 N/mm^2

The results obtained were recorded in Table 2 and Table 6.

4.2 Theoretical Force for Central Load W_1 = 200N

Using the method of joints, the theoretical force was calculated for W_1 = 100 N as illustrated in the appendix 8.1 for this report.

Using the Principle of Superposition, in order to calculate the values for the theoretical force for W_1 = 200 N, the results obtained calculated for W_1 = 100 N are simply doubled.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-200	-200	-200	173.2	173.2	173.2	173.2	-200	0	0	200	0	0

4.3 Theoretical Force for Central Load W_1 = 500N

Using the method of joints, the theoretical force was calculated for W_1 = 100 N as illustrated in the appendix 8.1 for this report.

Using the Principle of Superposition, in order to calculate the values for the theoretical force for W_1 = 500 N, the results obtained calculated for W_1 = 100 N are simply multiplied by 5.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-500	-500	-500	433	433	433	433	-500	0	0	500	0	0

4.4 Theoretical Force for Angled Load W_2 = 200N

Using the method of joints, the theoretical force was calculated for W_2 = 100 N as illustrated in the appendix 8.2 for this report.

Using the Principle of Superposition, in order to calculate the values for the theoretical force for W_2 = 200 N, the results obtained calculated for W_2 = 100 N are simply doubled.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-100	-100	-100	-86.6	-86.6	86.6	86.6	-100	0	0	100	-200	0

4.5 Theoretical Force for Angled Load $W_2 = 500N$

Using the method of joints, the theoretical force was calculated for W_2 = 100 N as illustrated in the appendix 8.2 for this report.

Using the Principle Of Superposition, in order to calculate the values for the theoretical force for W_2 = 500 N, the results obtained calculated for W_2 = 100 N are simply multiplied by 5.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-250	-250	-250	-216.5	-216.5	216.5	216.5	-250	0	0	250	-500	0

4.4 Theoretical Force for $W_1 = 500N$ and $W_2 = 200N$

Using the method of joints, the theoretical force was calculated as illustrated in the appendix 8.3 for this report.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-600	-600	-600	347	347	520	520	-600	0	0	600	-200	0

4.5 Theoretical Force for $W_1 = 500N$ and $W_2 = 500N$

Using the method of joints, the theoretical force was calculated as illustrated in the appendix 8.4 for this report.

The following results are obtained:

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Theoretical Force [N]	-750	-750	-750	217	217	650	650	-750	0	0	750	-500	0

5. RESULTS

5.1 For Central Load

Taking reference from the lab video provided, the values of strains in microstrains are noted in the table.

Table 1. Experimental Member Strains in Response to Central Load (W_1)

Load[N]	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	-18	-17	-17	19	20	19	19	-17	2	3	23	3	2
200	-35	-34	-34	35	36	34	35	-34	2	3	40	3	3
300	-52	-52	-52	50	51	49	49	-51	2	3	58	3	3
400	-70	-70	-70	65	67	64	64	-68	2	3	75	3	3
500	-87	-86	-86	80	81	78	78	-84	2	4	92	3	3

Using Formula (3) and the data from the above table (converted from microstrain to strain first) the values of experimental member force is obtained.

Table 2. Experimental Member Force in Response to Central Load (W_1)

Load[N]	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	-106.87	-100.93	-100.93	112.81	118.75	112.81	112.81	-100.93	11.87	17.81	136.56	17.81	11.87
200	-207.81	-201.87	-201.87	207.81	213.75	201.87	207.81	-201.87	11.87	17.81	237.50	17.81	17.81
300	-308.75	-308.75	-308.75	296.88	302.81	290.94	290.94	-302.81	11.87	17.81	344.38	17.81	17.81
400	-415.63	-415.63	-415.63	385.94	397.81	380.00	380.00	-403.75	11.87	17.81	445.32	17.81	17.81
500	-516.57	-510.63	-510.63	475.00	480.94	463.13	463.13	-498.75	11.87	23.75	546.26	17.81	17.81

The Experimental values are there after compared with the calculated Theoretical values.

Table 3. Comparison of Experimental and Theoretical Forces for Central Load, W_1= 200 N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-190	-187	-188	189	196	185	188	-184	13	17	219	15	15
Theoretical Force [N]	-200	-200	-200	173.2	173.2	173.2	173.2	-200	0	0	200	0	0

Table 4. Comparison of Experimental and Theoretical Forces for Central Load, W_1= 500 N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-473	-469	-471	436	444	425	428	-461	12	20	503	15	15
Theoretical Force [N]	-500	-500	-500	433	433	433	433	-500	0	0	500	0	0

5.2 For Angled Load

Taking reference from the lab video provided, the values of strains in microstrains are noted in the table.

Table 5. Experimental Member Strains in Response to Angled Load (W_2)

Load[N]	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	-6	-6	-6	-6	-5	4	5	-5	0	0	6	-13	0
200	-15	-14	-15	-16	-14	11	11	-12	0	0	15	-31	0
300	-24	-23	-23	-24	-21	17	18	-19	0	0	24	-48	0
400	-31	-30	-31	-32	-29	23	24	-25	0	1	31	-64	0
500	-40	-39	-40	-42	-38	30	31	-33	0	0	40	-82	0

Using Formula (3) and the data from the above table (converted from microstrain to strain first) the values of experimental member force is obtained.

Table 6. Experimental Member Force in Response to Angled Load (W_2)

Load[N]	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	-35.63	-35.63	-35.63	-35.63	-29.69	23.75	29.69	-29.69	0	0	35.63	-77.19	0
200	-89.06	-83.13	-89.06	-95.00	-83.13	65.31	65.31	-71.25	0	0	89.06	-184.07	0
300	-142.50	-136.57	-136.57	-142.50	-124.69	100.94	106.88	-112.81	0	0	142.50	-285.01	0
400	-184.07	-178.13	-184.07	-190.00	-172.19	136.57	142.50	-148.44	0	5.94	184.07	-380.01	0
500	-237.50	-231.57	-237.50	-249.38	-225.63	178.13	184.07	-195.94	0	0	237.50	-486.88	0

The Experimental values are there after compared with the calculated Theoretical values.

Table 7. Comparison of Experimental and Theoretical Forces for Angled Load, $W_2 = 200$ N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-84	-79	-81	-85	-75	61	62	-67	1	1	84	-168	1
Theoretical Force [N]	-100	-100	-100	-86.6	-86.6	86.6	86.6	-100	0	0	100	-200	0

Table 8. Comparison of Experimental and Theoretical Forces for Angled Load, $W_2 = 500$ N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-220	-213	-217	-227	-205	164	167	-180	-1	2	220	-447	2
Theoretical Force [N]	-250	-250	-250	-220	-216.5	216.5	216.5	-250	0	0	250	-500	0

5.3 For Superposition of Loads

We obtain the following values by adding values from Table 2 and Table 6.

Table 9. Superimposed Experimental Member Forces in Response to Combined Loads, $W_1 + W_2$

Load[N]	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
$W_1 = 0$ $W_2 = 0$	0	0	0	0	0	0	0	0	0	0	0	0	0
$W_1 = 500$ $W_2 = 0$	-516.57	-510.63	-510.63	475.01	480.95	463.13	463.13	-498.76	11.88	23.75	546.26	17.81	17.81
$W_1 = 500$ $W_2 = 100$	-552.20	-546.26	-546.26	439.38	451.26	486.88	492.82	-528.45	11.88	23.75	581.89	-59.38	17.81
$W_1 = 500$ $W_2 = 200$	-605.64	-593.76	-599.70	380.01	397.82	528.45	528.45	-570.01	11.88	23.75	635.32	-166.25	17.81
$W_1 = 500$ $W_2 = 300$	-659.07	-647.20	-647.20	332.51	356.26	564.07	570.01	-611.57	11.88	23.75	688.76	-267.19	17.81
$W_1 = 500$ $W_2 = 400$	-700.64	-688.76	-694.70	285.01	308.76	599.70	605.64	-647.20	11.88	29.69	730.33	-362.19	17.81
$W_1 = 500$ $W_2 = 500$	-754.08	-742.20	-748.14	225.63	255.32	641.26	647.20	-694.70	11.88	23.75	783.76	-469.07	17.81

The Experimental values are there after compared with the calculated Theoretical values.

Table 10. Comparison of Experimental and Theoretical Forces for Combined Loads, $W_1 = 500$N & $W_2 = 200$ N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-557	-548	-552	351	369	486	490	-528	13	21	587	-153	16
Theoretical Force [N]	-600	-600	-600	347	347	520	520	-600	0	0	600	-200	0

Table 11. Comparison of Experimental and Theoretical Forces for Combined Loads, $W_1 = W_2 = 500$ N

Member	AE	AG	AH	BE	BF	CI	CJ	DJ	EF	FG	GH	HI	IJ
Experimental Force [N]	-754.08	-742.20	-748.14	225.63	255.32	641.26	647.20	-694.70	11.88	23.75	783.76	-469.07	17.81
Theoretical Force [N]	-750	-750	-750	217	217	650	650	-750	0	0	750	-500	0

6. DATA ANALYSIS

After referring the data to the hypothesis made before conducting the experiment, it can be analysed the hypothesis made are considerably accurate.

There is no horizontal force H_B because at point B the truss is connected to a roller joint. It is known that a roller joint, in an ideal scenario, are free to rotate and translate along the surface its placed on. Since a roller joint is capable of horizontal translation it offers no reaction forces in that direction. Due to this reason, the roller joint is not able to provide any horizontal reaction forces. Hence, there is no H_B.

It can be noted that the magnitude of the true strains increase almost linearly as the frame is loaded i.e., the applied load is progressively increased.

The experimental and theoretical forces differ only slightly from each other. This difference is small compared to the order of magnitude of the forces themselves. This slight difference between the experimental and theoretical values, as mentioned in the hypothesis, probably arise due to experimental uncertainty/error as well as the number of assumptions made while carrying out the experiment. It can also be observed that in some cases this difference is much larger than in other cases. For instance, in the case of superposition of forces the discrepancies between experimental and theoretical is larger. This is possibly because the individual error from W_1 and W_2 was propagated during superposition and hence, the final result experienced a greater error. Another observation that can be made is that during the experimental procedure, the members that in theory were calculated to be zero members, i.e., members with zero reaction forces, did in fact experience small amounts of forces. This could possibly be because of error in experimentation or could also be cause due to subtle inaccuracies in the instruments used to carry out the experiment. These could also be caused by the lateral movement of the truss which is not considered when looking at a pin-joint. However, these forces were extremely small in comparison the other values and hence can be considered to be negligible. Since the pin-joint theory does not consider three-dimensional movement of the trusses and trusses are in reality more complex than the theory assumes, the exact behaviour of the trusses cannot be predicted using the simple pin-joint theory. However, this theory is representative of the general behaviour of the trusses.

On comparing the results obtained theoretically with the experimental results for the superposition of forces, we find that the values differ only slightly from each other. This implies that the results support the principle of superposition. In a case wherein the frame carries multiple loads, the total load of each of the members can be assessed by calculating these for each load (assuming no other load is present at the time) and then adding these values, in accordance with the principle of superposition to get the desired final result.

The simplifying assumptions made in the analysis include

- The weight of the structure is 0 N and the only load applied is the one at the nodes.
- All the members of the frame are connected by pin joints and so no moment is transferred on the members.
- Only tensile and compressive forces are taken into account. Moments and torque are neglected.
- All Joints are frictionless

The member HI is peculiarly of special interest. This can be attributed to the fact that when the truss is attributed to only a downward force, it acts as a zero member. However, when an angled load is acting on the truss, it experiences a certain amount of force hence indicating a change in its behaviour. This brings to light an important aspect of analysing trusses – it is essential to study its behaviour not only under different magnitudes of forces but also different types acting on different points of a truss as it was clearly proven how a truss and its members act differently under different conditions. It is thus important that we examine all of the load cases that a structure may be exposed to in its analysis.

The members EF and IJ act as zero members under all conditions. This means that they do not experience any force regardless of the magnitude and direction of the force applied to the main framework. Thus, in theory these members are not essential to the structure of the truss. However, in the event of breakdown of any other member of the framework, the members EF and IJ could possibly help bear the load and prevent breakdown of the truss completely or at least provide a sufficient window to repair the structure without causing much serious damage.

Tension cables can possibly replace members that are under tension. However, the horizontal members (BE, BF, CI and CJ) are in tension but are perpendicular to the central load and hence end up inducing a curve on application of force. To avoid this it is more appropriate to use members parallel to direction of applied load and in tension as tension cables (because cables are able to resist force on the axis parallel to their orientation). Thus elements EF, GH, IJ and EF can be potentially replaced by cables. This replacement will undoubtedly reduce the overall weight of the structure but at the same time it will decrease the general stability of the structure.

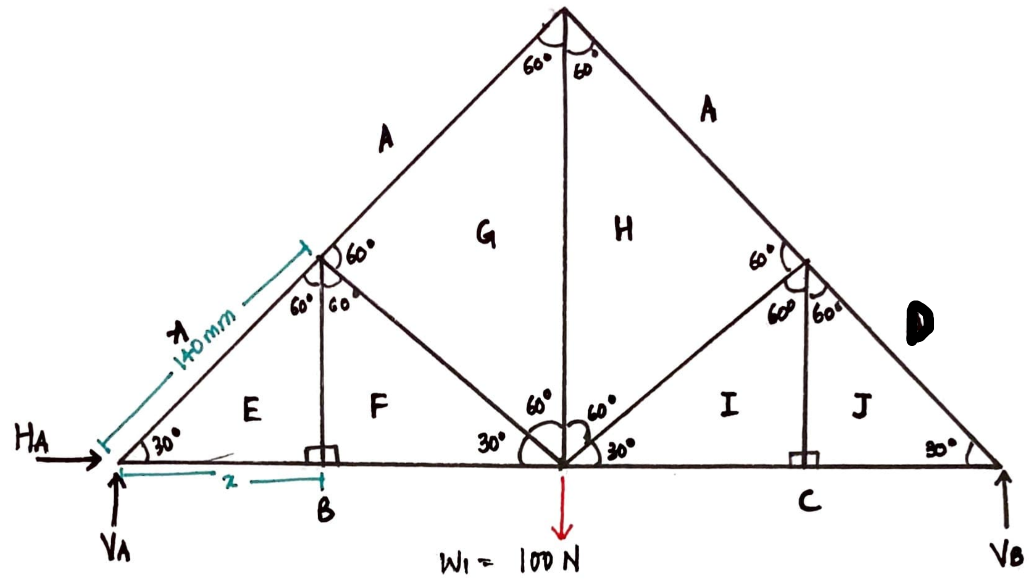
7. CONCLUSION

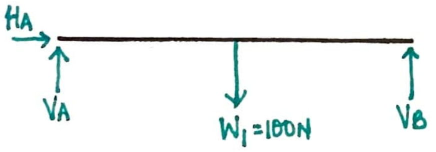
Within the scope of experimental error, the experiment can be considered to be successful as its aims were achieved and reasonably useful observations and insights were made from the data gathered through it. The effect of changing magnitudes and directions of force on the framework was studied. This led to numerous observations like the members under tension, compression and the zero members under varied conditions. It was also noted that as the applied force increases the strain and force in the members also increases linearly. Many assumptions were made for the sake of simplification but their impact on the resultant values was also assessed. Another important result of the experiment was that it was realised that it is essential to examine all the load cases that a structure may be exposed to in its analysis as different members exhibit different behaviours when exposed to different conditions of force. The experimental and the theoretical values of the forces calculated were also similar and hence this helped assess the accuracy of obtaining results via using the calculating method. The principal of superposition for calculation of forces in a truss was also confirmed through this experiment and it may be used to calculate the force in the elements of a truss subject to multiple loads. The experiment as a whole was found to be insightful and useful.

8. APPENDIX

8.1 Calculation for Downward Force W_1 = 100N using Method of Joints

Downward Force





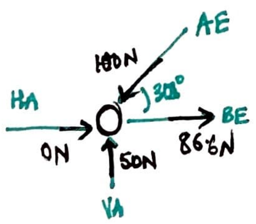
$\uparrow^{+}$ $V_A + V_B - 100 = 0$

$\rightarrow^{+}$ $H_A = 0$

$\circlearrowright^{+}$ $2x(100) - 4x(V_B) = 0$

$\Rightarrow V_B = 50N$

$\therefore V_A = 50N$



$\uparrow^{+}$ $V_A + AE\sin 30° = 0$

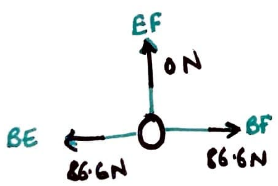
$AE = -50(2)$

$AE = -100N$

$\rightarrow^{+}$ $H_A + BE + AE\cos 30° = 0$

$\Rightarrow BE = 100\cos 30° - 0$

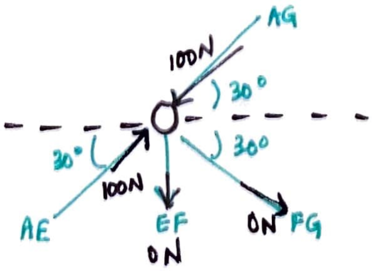
$\Rightarrow BE = 86.6N$



$\uparrow^{+}$ $EF = 0N$

$\rightarrow^{+}$ $BF - BE = 0$

$\Rightarrow BF = 86.6N$



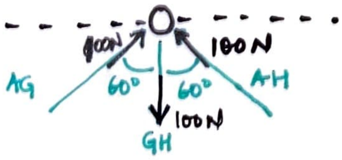
$\uparrow^{+}$ $AG\sin 30° + AE\sin 30° - EF - FG\sin 30° = 0$

$\Rightarrow AG - FG = -100$

$\rightarrow^{+}$ $AE\cos 30° + AG\cos 30° + FG\cos 30° = 0$

$\Rightarrow AG + FG = -100$

Thus $AG = -100N$; $FG = 0N$



$\uparrow^{+}$ $AG\cos 60° - GH - AH\cos 60° = 0$

$\rightarrow^{+}$ $AG\sin 60° + AH\sin 60° = 0$

$\Rightarrow AH = -100N$

Putting value in above equation

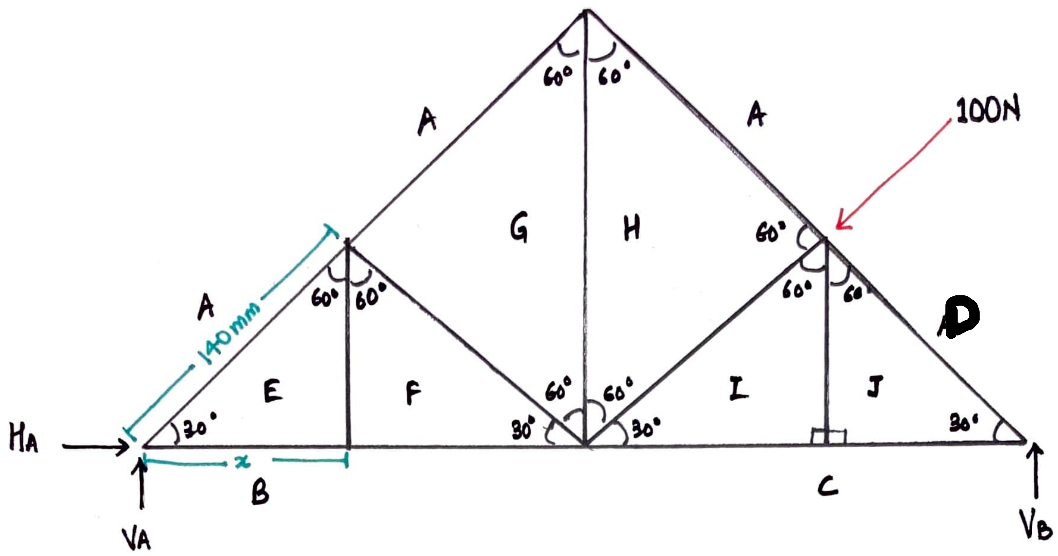
$GH = (AG - AH)\cos 60°$

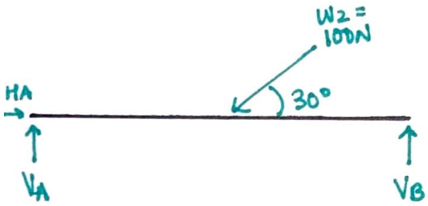
$\Rightarrow GH = 100N$

Due to Symmetry the values on the left and right of member GH are the same and hence can be easily determined from the calculations made above.

8.2 Calculation for Angled Force W_2 = 100N using Method of Joints

Slanting Force





$\uparrow^+ \quad V_A + V_B - 100\sin 30° = 0$

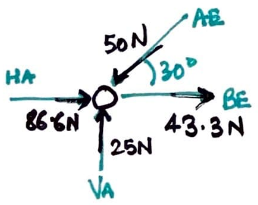
$\rightarrow^+ \quad H_A - 100\cos 30° = 0$

$\Rightarrow H_A = 86.6N$

$\circlearrowright^+ \quad \cancel{2x}(100\sin 30°) - \overset{2}{\cancel{4x}}(V_B) = 0$

$\Rightarrow V_B = 25N$

$\therefore V_A = 25N$



$\uparrow^+ \quad V_A + AE\sin 30° = 0$

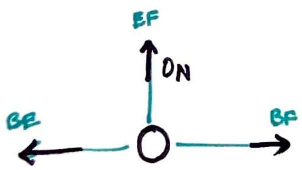
$\Rightarrow AE = -25(2)$

$\Rightarrow AE = -50N$

$\rightarrow^+ \quad H_A + BE + AE\cos 30° = 0$

$\Rightarrow BE = 50\cos 30° - 86.6$

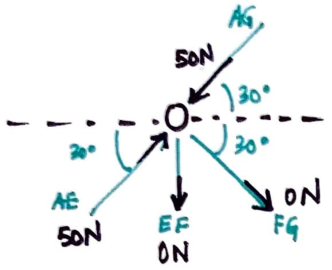
$\Rightarrow BE = -43.3N$



$\uparrow^+ \quad EF = 0N$

$\rightarrow^+ \quad BF - BE = 0$

$\Rightarrow BF = -43.3N$



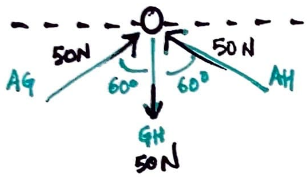
$\uparrow^{+}$ $AG \sin 30° + AE \sin 30° - EF - FG \sin 30° = 0$

$\Rightarrow AG - FG = -50$

$\rightarrow^{+}$ $AE \cos 30° + AG \cos 30° + FG \cos 30° = 0$

$\Rightarrow AG + FG = -50$

Thus $AG = -50N$, $FG = 0N$



$\uparrow^{+}$ $AG \cos 60° - GH - AH \cos 60° = 0$

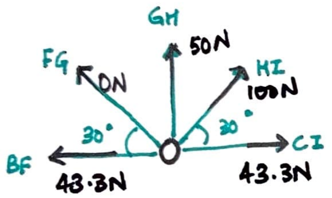
$\rightarrow^{+}$ $AG \sin 60° + AH \sin 60° = 0$

$\Rightarrow AH = -50N$

Putting value in above eq^n

$GH = (AG - AH) \cos 60°$

$\Rightarrow GH = 50N$



$\uparrow^{+}$ $GH + FG \sin 30° + HI \sin 30° = 0$

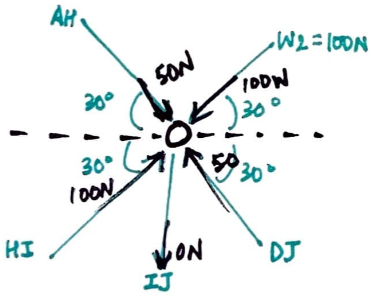
$\Rightarrow HI \sin 30° = -50$

$\Rightarrow HI = -100N$

$\rightarrow^{+}$ $-BF + CI + HI \cos 30° - FG \cos 30° = 0$

$\Rightarrow -43.3 + CI - 100 \cos 30° = 0$

$\Rightarrow CI = 43.3N$



$\uparrow^{+}$ $HI \sin 30° - AH \sin 30° - W_2 \sin 30° - DJ \sin 30° - IJ = 0$

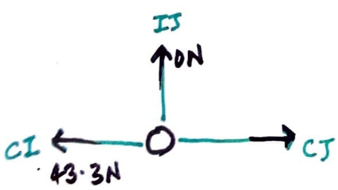
$\Rightarrow -50 \sin 30° - DJ \sin 30° - IJ = 0$

$\rightarrow^{+}$ $AH \cos 30° + HI \cos 30° + DJ \cos 30° - W_2 \cos 30° = 0$

$\Rightarrow DJ = -50N$

Putting value in above eq^n

$\Rightarrow IJ = 0N$

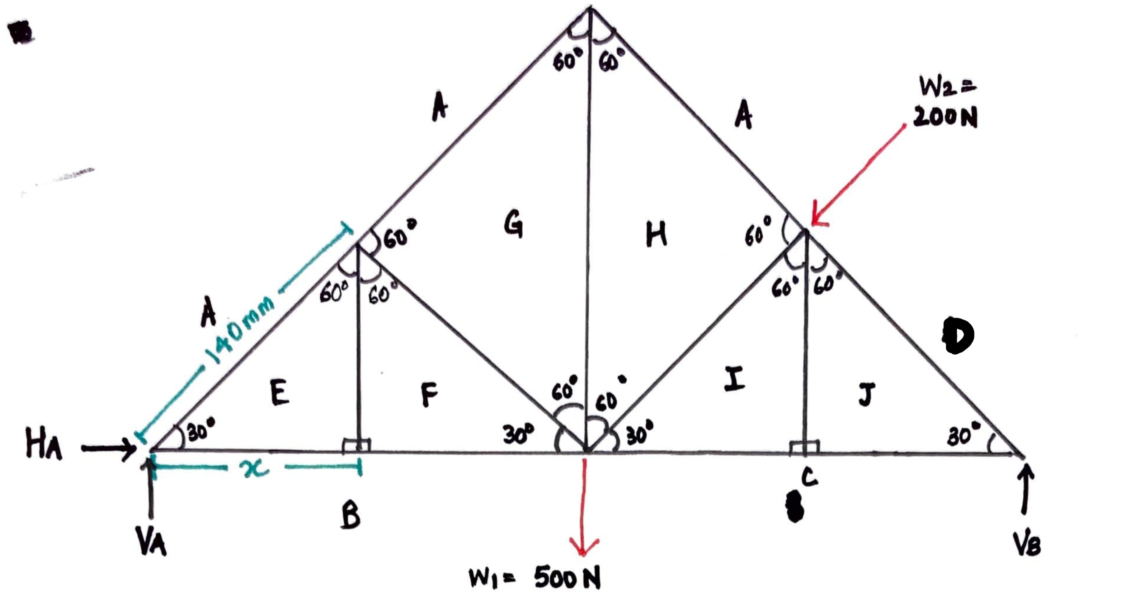


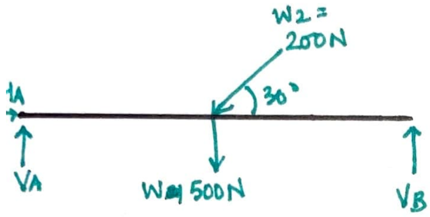
$\rightarrow^{+}$ $-CI + CJ = 0$

$\Rightarrow CJ = 43.3N$

8.3 Calculation for Combined Force $W_1 = 500N$ and $W_2 = 200N$ using Method of Joints

Combined Forces





$$\uparrow^{+}\ V_A + V_B - 500 - 200\sin 30^\circ = 0$$

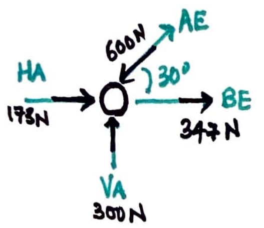
$$\xrightarrow{+}\ H_A - 200\cos 30^\circ = 0$$

$$\Rightarrow H_A = 173N$$

$$\circlearrowleft^{+}\ 2x(500) + 2x(200\sin 30^\circ) - 4x(V_B) = 0$$

$$\Rightarrow V_B = 300N$$

$$\therefore V_A = 300N$$



$$\uparrow^{+}\ V_A + AE\sin 30^\circ = 0$$

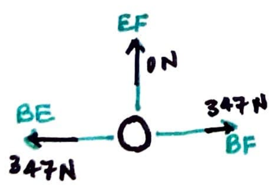
$$\Rightarrow AE = -300(2)$$

$$\Rightarrow AE = -600N$$

$$\xrightarrow{+}\ H_A + BE + AE\cos 30^\circ = 0$$

$$\Rightarrow BE = 600\cos 30^\circ - 173$$

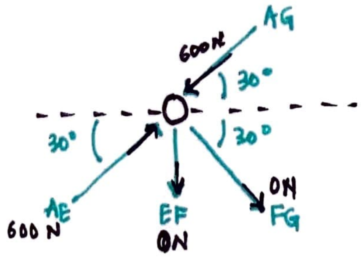
$$\Rightarrow BE = 347N$$



$$\uparrow^{+}\ EF = 0N$$

$$\xrightarrow{+}\ BF - BE = 0$$

$$\Rightarrow BF = 347N$$



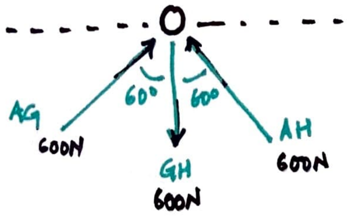
$\uparrow^{+}$ $AG\sin 30° + AE\sin 30° - EF - FG\sin 30° = 0$

$\Rightarrow AG - FG = -600$

$\rightarrow^{+}$ $AE\cos 30° + AG\cos 30° + FG\cos 30° = 0$

$\Rightarrow AG + FG = -600N$

Thus $AG = -600N$; $FG = 0N$



$\uparrow^{+}$ $AG\cos 60° - GH - AH\cos 60° = 0$

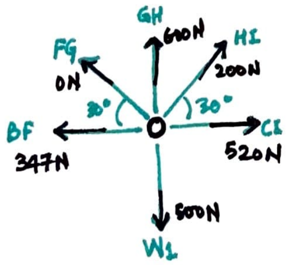
$\rightarrow^{+}$ $AG\sin 60° + AH\sin 60° = 0$

$\Rightarrow AH = -600N$

Putting value in above eqn

$GH = (AG - AH)\cos 60°$

$\Rightarrow GH = 600N$



$\uparrow^{+}$

$-W_1 + GH + FG\sin 30° + HI\sin 30° = 0$

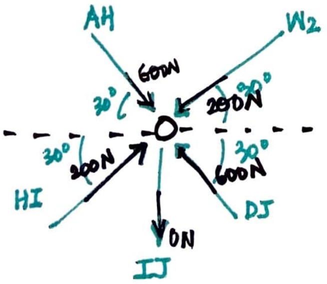
$\Rightarrow -500 + 600 = -HI\sin 30°$

$\Rightarrow HI = -200N$

$\rightarrow^{+}$ $-BF + CI + HI\cos 30° - FG\cos 30° = 0$

$\Rightarrow -347 + CI - 200\cos 30° = 0$

$\Rightarrow CI = 520N$



$\uparrow^{+}$ $HI\sin 30° - AH\sin 30° - W_2\sin 30° - DJ\sin 30° - IJ = 0$

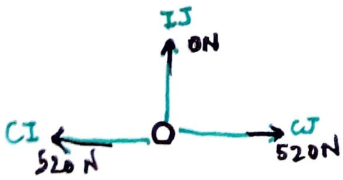
$\Rightarrow -600\sin 30° - DJ\sin 30° - IJ = 0$

$\rightarrow^{+}$ $AH\cos 30° + HI\cos 30° + DJ\cos 30° - W_2\cos 30° = 0$

$\Rightarrow DJ = -600N$

Putting value in above eqn

$\Rightarrow IJ = 0N$

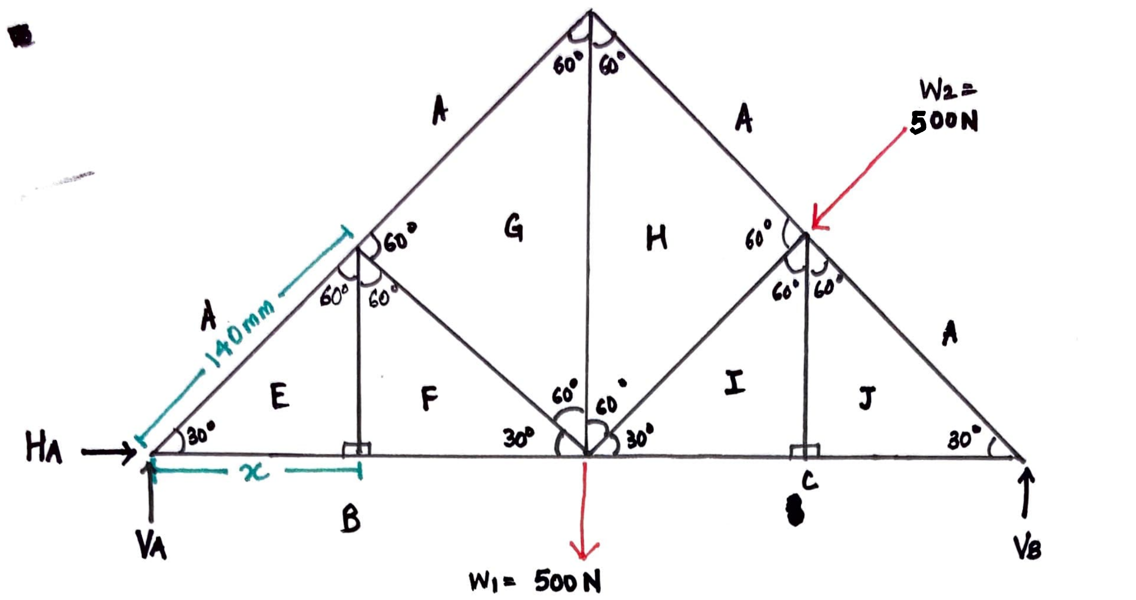


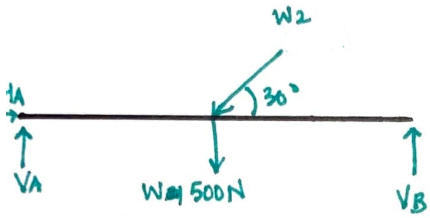
$\rightarrow^{+}$ $-CI + CJ = 0$

$\Rightarrow CJ = 520N$

8.4 Calculation for Combined Force $W_1 = 500N$ and $W_2 = 500N$ using Method of Joints

Combined Forces





$\uparrow^{+}$ $V_A + V_B - 500 - 200 \sin 30^\circ = 0$

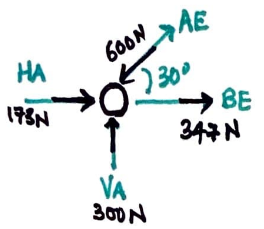
$\rightarrow^{+}$ $H_A - 200 \cos 30^\circ = 0$

$\Rightarrow H_A = 173N$

$\circlearrowleft^{+}$ $2x(500) + 2x(200 \sin 30^\circ) - 4x(V_B) = 0$

$\Rightarrow V_B = 300N$

$\therefore V_A = 300N$



$\uparrow^{+}$ $V_A + AE \sin 30^\circ = 0$

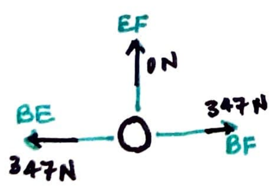
$\Rightarrow AE = -300(2)$

$\Rightarrow AE = -600N$

$\rightarrow^{+}$ $H_A + BE + AE \cos 30^\circ = 0$

$\Rightarrow BE = 600 \cos 30^\circ - 173$

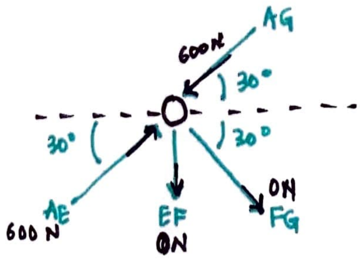
$\Rightarrow BE = 347\,N$



$\uparrow^{+}$ $EF = 0N$

$\rightarrow^{+}$ $BF - BE = 0$

$\Rightarrow BF = 347N$



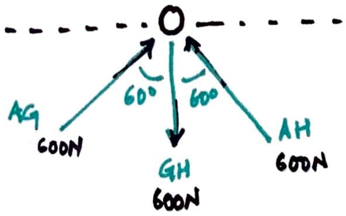
$\uparrow^{+}$ $AG \sin 30° + AE \sin 30° - EF - FG \sin 30° = 0$

$\Rightarrow AG - FG = -600$

$\rightarrow^{+}$ $AE \cos 30° + AG \cos 30° + FG \cos 30° = 0$

$\Rightarrow AG + FG = -600N$

Thus $AG = -600N$; $FG = 0N$



$\uparrow^{+}$ $AG \cos 60° - GH - AH \cos 60° = 0$

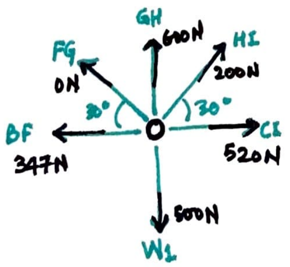
$\rightarrow^{+}$ $AG \sin 60° + AH \sin 60° = 0$

$\Rightarrow AH = -600N$

Putting value in above eq^n

$GH = (AG - AH) \cos 60°$

$\Rightarrow GH = 600N$



$\uparrow^{+}$

$-W_1 + GH + FG \sin 30° + HI \sin 30° = 0$

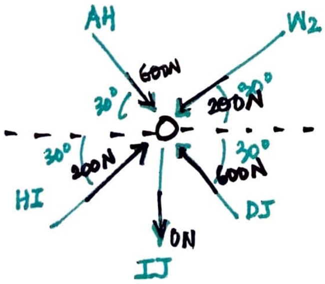
$\Rightarrow -500 + 600 = -HI \sin 30°$

$\Rightarrow HI = -200N$

$\rightarrow^{+}$ $-BF + CI + HI \cos 30° - FG \cos 30° = 0$

$\Rightarrow -347 + CI - 200 \cos 30° = 0$

$\Rightarrow CI = 520N$



$\uparrow^{+}$ $HI \sin 30° - AH \sin 30° - W_2 \sin 30° - DJ \sin 30° - IJ = 0$

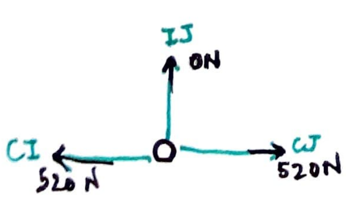
$\Rightarrow -600 \sin 30° - DJ \sin 30° - IJ = 0$

$\rightarrow^{+}$ $AH \cos 30° + HI \cos 30° + DJ \cos 30° - W_2 \cos 30° = 0$

$\Rightarrow DJ = -600N$

Putting value in above eq^n

$\Rightarrow IJ = 0N$



$\rightarrow^{+}$ $-CI + CJ = 0$

$\Rightarrow CJ = 520N$